COLLAPSING STARS

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1 - INTRODUCTION

The initial element in GR is a differentiable manifold U having four dimension and of class C^2.

At each point of U, there exits a metric tensor g , this gives to U a normal hyperbolic riemannian structure.

If we now suppose that gravity can be discribed as a field of symmetrical connexion, one can then proceed to demontrate that this field is coincident with the riemannian connexion of (U,g). In other words one can consider the components of tensor g as being potentials of the gravity field. It also follows that the trajectories of a test particle become geodesics of the riemannian manifold (U,g).

If we consider in U a perfect fluid we are led to the einsteinian equations for the interior case in which the second member is the stress-energy tensor of the fluid.

A perfect fluid possesses the following fondamental property : the current lines are geodesics of the riemannian manifold (U,γ) where $\gamma = F^2 g$, F is the index of the fluid , ([1] p. 71 to 83).

The consideration of the metric tensor γ leads to a new definition of the time, the following two paragraphs resume the essential results of Mathé ([2], [3], [4], [5] and [6]). Geometric units in which the speed of light c is equal to unity are used.

2 - PHYSICAL DEFINITION OF TIME

Consider U containing a perfect fluid with an equation of state linking the density ρ of the fluid and its pressure p. If the fluid is taken as being irrotational, it can be studied in comoving co-ordinate systems as follows:

$$g = e^{2\omega} dt^2 - h_{ij}\, dx_i\, dx_j \qquad (i , j = 1, 2, 3)$$

where h_{ij} is the defined positive metric tensor of space ; p, ρ, ω, h_{ij} functions of (t, x_1, x_2, x_3). The energy-impulse tensor of the fluid is expressed as follows:

$$T_0^{\ 0} = \rho \,; \quad T_1^{\ 1} = T_2^{\ 2} = T_3^{\ 3} = -p$$

The conservation identities give:

$$\partial_i p + (\rho + p)\, \partial_i \omega = 0 \qquad 1 \leq i \leq 3 \tag{1}$$

$$\partial_0 \rho + (\rho + p)\, \partial_0 (\ln\sqrt{h}) = 0 \tag{2}$$

where h stands for the determinant of h_{ij}. When considering F, the index of the fluid:

$$F = \mathrm{Exp}\left(\int dp / (\rho + p) \right) \tag{3}$$

(1) proves that Fe^{ω} is independent of x_1, x_2 et x_3, in other words Fe^{ω} is a function of t only. According to Lichnerowicz ([1] p.75) however, the flow lines of the fluid are geodesics of the metric:

$$\gamma = F^2 g = F^2 e^{2\omega} dt^2 - F^2 h_{ij}\, dx_i\, dx_j$$

Consequently, the coefficient of dt^2 within γ is <u>uniquely function of t</u> and a simple change of time-scale gives:

$$d\tau = F e^{\omega} dt \tag{4}$$

it is possible to write :

$$\gamma = d\tau^2 - F^2 h_{ij}\, dx_i\, dx_j \tag{5}$$

where F et h_{ij} now become functions of (τ, x_1, x_2, x_3).

The metric tensor γ is the frame of the evolution of the fluid. Time τ is thus defined in a univocal manner and should be chosen as an absolute time. This term does not mean a return to Newton's absolute time. The expression "cosmic time" would be suitable but it is usually reserved for time t.

(2) and (3) further give:

$$\partial_0 (\rho + p)/(\rho + p) = \partial_0 F / F - \partial_0 (\sqrt{h}) / h \tag{6}$$

which shows:

$$(\rho + p)\sqrt{h} / F = C(x_1, x_2, x_3) \tag{7}$$

3 - APPLICATION TO A COLLAPSING STAR

We suppose that the star studied be spherical, be made of "perfect fluid" . For its study we consider two cases :

For the exterior case we apply the Birkhoff's theorem :

Let the geometry of a given region of spacetime be spherically symmetric and be solution to the Einstein field equations in vacuum. Then that geometry is necessarily a piece of the Schwarzschild geometry.

For the interior case, we describe the collapsing star with comoving co-ordinate systems as the Friedmann's model (with $r \to r / R_0$). We suppose that the mass is constant. We write the metric tensor in the form :

$$g = dt^2 - (R^2 / R_0^2)\, dl^2 \qquad (8)$$

where R is a function of t and :

$$dl^2 = dr^2 / (1 - k\, r^2 / R_0^2) + r^2 (d\theta^2 + \sin^2\theta\, d\varphi^2) \qquad (9)$$

where k = -1, 0 or 1 according to whether the curvature of the universe is negative, null or positive. We have the Einstein's equations.

The initial values, for t = 0, will be index with zero and we set : $S = R / R_0$.

$$3 ((dS/dt)^2 + k / R_0^2) / S^2 = 8\pi \rho \qquad (10)$$

$$2 (d^2S/dt^2) / S + (dS/dt)^2 / S^2 + k / (R_0^2 S^2) = - 8\pi p \qquad (11)$$

equation (11) can be replaced by the equation of conservation deduced from (7).

$$(\rho + p)\, S^3 / F = (\rho_0 + p_0) / F_0 = Cst \qquad (12)$$

According to (3) F is a function of t and (4) gives the absolute time as being:

$$d\tau = F\, dt \qquad (13)$$

(5) gives:

$$\gamma = F^2 g = d\tau^2 - Q^2 dl^2 \qquad (14)$$

where the observed radius of the star is:

$$Q = FR / R_0 = FS \qquad (15)$$

If " $'$ " is taken as the derivation with respect to τ we get for any derivable function f:

$$df/dt = Ff' \qquad (16)$$

The quantities which characterise the fluid of the star should be studied as function of τ and not of t. Using the equation of state and substituting in (3) and (12) we obtain ρ then F as a function of S. Further, (16) gives:

$$dS/dt = FS'$$

the equation (10) then becomes:

$$S'^2 = (- k + 8\pi \rho S^2/3R_0{}^2) / F^2 \qquad (17)$$

the collapse begins for t = 0 therefore we have: $S'_0 = 0$, (17) gives :

$$k = 8\pi\rho_0 / 3R_0^2 = 1$$

therefore the curvature is necessarily positive. The equation (17) becomes :

$$S'^2 = ((\rho/\rho_0) S^2 - 1) / F^2 \qquad (18)$$

Integration of this differential equation gives the variation of S as a function in τ. When equation (3), (12) and (15) permit it, ρ, p, F and R can be expressed as a function of Q. Substituting in (18) gives the differential equation that verifies Q. Beyond the fact that is not always possible, the differential equation for Q is often more complicated than that for S. On the other hand, once (17) has been resolved, substituting for S in ρ, p, F and Q gives the variation of these quantities as a function of the time τ. Then the equation (13) gives the expression of t as a function in τ.

$$t = \int (d\tau / F) \qquad (19)$$

4 - EQUATION OF STATE FOR A COLLAPSING STAR

We consider only the case of a star with density superior to that of nuclear matter (3.6 10^{14} g/cm^3), the first thing to do is to determine the equation of state of the perfect fluid of the star.

In 1961 Levinger and Simmons ([7] and [8]) investigated two potentials V_β and V_γ to describe the strong interaction of nucleous. V_β is a square-well potential with a tail of the Yukawa type and V_γ is a combination of exponentially decreasing terms. Both of these potentials lead to similar results and, for densities greater than 10^{15} g/cm^3, give a composite equation of state for matter :

$$\mathbf{p = \rho}$$

this equation of state corresponds to the Lichnerowicz's case of incompressibility ([1] p.91), note here that the result of the virial theorem $p < \rho / 3$ is inapplicable to very high densities because of strong interaction.

5- RESOLUTION OF THE EQUATIONS

The relations (3), (12), (15) and (18) give :

$$F = F_0 (\rho/\rho_0)^{1/2} = F_0 / S^3 \tag{20}$$

$$\rho = \rho_0 / S^6 \tag{21}$$

$$Q = F_0 / S^2 \tag{22}$$

$$S'^2 = S^2 (1 - S^4)/ F_0^2 \tag{23}$$

$$Q'^2 = 4 ((Q / F_0)^2 - 1) \tag{24}$$

The integration of the equation is immediate and we obtain :

$$Q = F_0 \cosh (2 \tau / F_0) \tag{25}$$

$$S = \cosh^{-1/2} (2 \tau / F_0) \tag{26}$$

6 - CONCLUSION

The expressions of S and Q shows that, if we describe the interior case of a collapsing star with the only scale of absolute time in GR, i.e. with the metric tensor $\gamma = F^2 g$, we obtain a result without singularity because we have:

$S (\tau) > 0$ and $Q (\tau) > 0$ for $\tau > 0$

Note here that for an interior observer, if it is possible, the radius of the star, Q, always increases. As to an exterior case the collapse is going on infinitely ([9] p. 846 to 850).

In all cases the much talked-of black hole never appears.

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